Higher Dimensional Dissipative Future Universe Without Big Rip

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Abstract— Higher dimensional dissipative future universe without Big rip in the context of Ecart formalism has been studied. In this work, we have studied the generalized Chaplygin gas characterized by equation of state $p=-\frac{A}{\rho^{1/\alpha}}$ as a model for dark energy. We have verified that in higher dimensions when the cosmic dark energy behaves simultaneously like a fluid with equation of state $p=\omega\rho$; $\omega<-1$ as well as Chaplygin gas then big rip does not arise and the scale factor is found to be regular for all time. The work of Yadav (2011) has been extended and studied in higher dimensions.

Keywords : Phantom fluid, Big rip, Accelerated universe, Higher dimensions.

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1 INTRODUCTION

RECENT Ia Supernova observations indicate that the current universe is not only expanding but also accelerating. This behavior of universe is confirmed by various independent observational data, the large scale structure, the cosmic microwave background (CMB) radiation and so on. There is consensus on conclusion that the universe has entered a state of accelerating expansion at red shift $z \approx 0.5$. These recent observations strongly indicate that our universe is spatially flat and there exists an exotic fluid called as dark energy with negative pressure.
Most of the perfect fluids relevant to cosmology obey an equation of state of the form

$$p=\omega\rho \ ,$$

where ω is the equation of state parameter.
The three most common examples of cosmological fluids with constant ω are

(i) Dust model $(\omega=0)$,

(ii) Radiating model $\left(\omega=1/3\right)$ and

(iii) Vacuum energy model $(\omega=-1)$ which is mathematically equivalent to cosmological constant $\wedge$.

It is well known that fluids with $\omega<-\frac{1}{3}$ are usually considered in the context of dark energy (DE), since they give rise to accelerating expansion. As per Coroll *et. al.* [11] the fluid with constant EOS parameter with $\omega<-\frac{1}{3}$, particularly in constant of DE there have been proposed various scalar field models that can be described by time dependent ω and that can evolve below $-\frac{1}{3}$. E.g. quintessence $(-1\le\omega\le 1)$, Phantom $(\omega\le -1)$, quintom that evolve across cosmological constant boundary $\omega=-1$. However, we still do not have a perfect understanding of EOS of the DE and some time it is useful to think about the Einstein field equations without specifying the theory from which EOS parameter ω is derived (Jaffe *et. al.* [19]). The astrophysical observations also indicate that the universe media is not a perfect fluid (Brevik *et. al.* [5]) and the viscosity is considered in the evolution of the universe by (Brevik *et. al.* [6], Caldwell et. al. [7] and Cataldo, et. al. [8]). On the other hand, in the standard cosmological model if EOS parameter ω is less than -1 the universe shows the future finite singularity called Big rip (Jackiw [18], Nojiri *et. al.* [23]).
Cosmologists and particle physicists have considerable interest in obtaining solutions of Einstein's equations in higher dimensions in the context of physics of early universe. In its early stage, it is well known that the universe was much small than it is today. Cosmologists are interested in theories with more than four space-time dimensions in which extra dimensions are connected to a very small size beyond our present ability of experimental detection (Krori and Barua [21]). The experimental detection of time variation of fundamental constants could provide strong evidence for the existence of extra dimensions. (Chodos and Detweller [10], Marciano [22]) proposed cosmological dimensional reduction process such that the five dimensional universe naturally evolves into four dimensional universe as a consequence of dimensional reduction. A number of authors ([1], [2], [9], [20], [26]] have studied higher dimensional cosmological models containing variety of matter fields.
In Roberson Walker cosmology, the EOS for Chaplygin gas is given by

$$p=-\frac{A}{\rho} \ , \tag{1}$$

where p is pressure and ρ is energy density in co-moving reference frame with $\rho>0$ and A is positive constant.
The Chaplygin gas has super symmetry generalization (Bento *et. al.* [3], Gorini *et. al.* [15]). Betrolami *et. al.* [4] have found that generalized Chaplygin gas (GCG) is better fit for latest Supernova data.
For generalized Chaplygin gas, the EOS is given by,

$$p = -\frac{A}{\rho^{1/\alpha}}\ , \quad \text{with } 1 \le \alpha < \infty .$$ (2)

In particular for $\alpha = 1$, generalized Chaplygin gas reduces to Chaplygin gas.

Zhai et. al. [28] have investigated a viscous generalized Chaplygin gas.. Szydlowsk et. al. [25] found that dissipative Chaplygin gas can give rise to structurally stable evolutional scenario. It is interesting to note that generalized Chaplygin gas itself can behave like a fluid with viscosity in the context of Ecart Formalism. Fabris *et al.* [14] have investigated the equivalence generalized Chaplygin gas and dust like fluid. Recently Cruz *et al*. [12] have studied dissipative generalized Chaplygin gas as Phantom dark energy and established the cosmological solutions to generalized Chaplygin gas with bulk viscosity.

In this paper, we consider higher dimensional FRW metric for homogeneous and isotropic flat universe. The work of Yadav [27] is extended in five dimensional space times.

2. FIELD EQUATIONS

The higher dimensional FRW metric for homogeneous and isotropic flat universe is given by

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2 + dv^2\right), \qquad (3)$$

where *a(t)* is a scale factor and *t* represents the cosmic time; v is the fifth dimension in space. Here the suffixes 0, 1, 2, 3, 4 represent the variables *t, x, y, z, v* respectively.

The energy momentum tensor for bulk viscous cosmology in early universe is given by

$$T_j^i = (\rho + \bar{p})\, u^i u_j + \bar{p} g_j^i$$ (4)

with $$\bar{p} = p - 3\xi H\ ,$$ (5)

where $\rho \rightarrow$ energy density, $p \rightarrow$ isotropic pressure, $\bar{p} \rightarrow$ effective pressure, $\xi \rightarrow$ bulk viscous coefficient, $H \rightarrow$ Hubble's parameter, $g_{ij} \rightarrow$ metric tensor. u^i is the four velocity of fluid which satisfy the condition $g_{ij}u^i u^j = -1$.

Using the above equations, the matter tensor is given by ,

$$T_j^i = diag.(-\rho, \bar{p}, \bar{p}, \bar{p}, \bar{p})$$ (6)

The Einstein's Field equations are

$$R_j^i - \frac{1}{2} g_j^i R = -T_j^i$$ (7)

Where $R_{ij} \rightarrow$ Ricci Tensor,

$R \rightarrow$ Ricci Scalar,

$T_{ij} \rightarrow$ Energy momentum tensor for bulk viscous cosmology.

Using the equations (4), (5) and (6) for metric (3), the Einstein's Field equations (7) reduce to

$$\frac{\dot{a}^2}{a^2} = \frac{\rho}{6} = H^2$$ (8)

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 2\bar{p})$$ (9)

Here (.) dot represents the differentiation with respect to *t*.

The energy conservation equation is given as,

$$T^{ij}_{;j} = 0 \quad \text{where} \quad T^{ij}_{;j} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}\left[T^{ij}\sqrt{-g}\right] + T^{jk}\,\Gamma^i_{jk}$$

Using Equation (3) and (6) which simplifies to

$$\dot{\rho} + 4H(\bar{p} - \rho) = 0\ ,$$ (10)

where $\dot{\rho}$ is the differentiation of ρ with respect to *t*.

3. SOLUTION OF THE FIELD EQUATIONS

Using equations (2), (5), (8), equation (10) can be written as

$$\dot{\rho} + 4\frac{\dot{a}}{a}\left[\rho - \frac{A}{\rho^{1/\alpha}} - 3\xi H\right] = 0\ .$$ (11)

In most of the investigations in cosmology, the viscosity is assumed to be a simple power law function of the energy density (Shri Ram [24], Yadav [27]) *i.e*

$$\xi = \xi_0 \rho^n\ ,$$ (12)

where ξ_0 and n are constants.

From equations (11) and (12) we get,

$$\frac{d\rho}{dt} + 4\frac{da}{dt}\left[\frac{\rho^{\alpha+1/\alpha} - A}{\rho^{1/\alpha}}\right] = 2\xi_0\rho^{n+1}\ . \qquad (13)$$

Using the transformation $\frac{d\rho}{dt} = f(\rho) = \rho^{n+1}$ in equation (13) which reduces to,

$$\rho^{1+\alpha/\alpha} = A + \left(\rho_0^{1+\alpha/\alpha} - A\right)\left(\frac{a_0}{a}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}}, \qquad (14)$$

where ρ_0, a_0 represent the values of $\rho(t)$ and $a(t)$ at present time t_0 respectively.

The dark energy simultaneously behaves like GCG obeying equation (2) as well as fluid with equation of state.

$$p = \omega\rho\ , \text{ with } \quad \omega < -1\ .$$ (15)

From equations (2) and (15) we get,

$$\omega(t) = -\frac{A}{\rho^{1+\alpha/\alpha}}\ .$$ (16)

At $t = t_0$ equation (16) gives

$$A = -\omega_0 \rho_0^{\frac{1+\alpha}{\alpha}}\ ,$$ (17)

where ω_0 is value of $\omega = \omega(t)$ at $t = t_0$.

Put the value of *A* from equation (17) in equation (14) we get,

$$\rho = \rho_0 \left[-\omega_0 + (1+\omega_0)\left(\frac{a_0}{a}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}} \right]^{\alpha/1+\alpha} . \tag{18}$$

In homogeneous model of universe, scalar field $\phi(t)$ with potential $V(\phi)$ has energy density

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{19}$$

and $$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{20}$$

where $\dot{\phi}$ is differentiation of ϕ with respect to *t*.
Adding (19) and (20), we get

$$\dot{\phi}^2 = \rho_\phi + p_\phi . \tag{21}$$

With the help of equations (2) and (17), equation (21) gives

$$\dot{\phi}^2 = \frac{\rho^{1+\alpha/\alpha} + \rho_0^{1+\alpha/\alpha}\,\omega_0}{\rho^{1/\alpha}} . \tag{22}$$

Substitute the value of ρ from equation (18) in equation (22) we get,

$$\dot{\phi}^2 = \frac{(1+\omega_0)\rho_0 \left(\frac{a_0}{a(t)}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}}}{\left[-\omega_0 + (1+\omega_0)\left(\frac{a_0}{a(t)}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}}\right]^{1/1+\alpha}} . \tag{23}$$

From equation (23), it is observed that when $\dot{\phi}^2 > 0$, we get positive kinetic energy for $(1+\omega_0) > 0$ and when $\dot{\phi}^2 < 0$, we get negative kinetic energy for $(1+\omega_0) < 0$.
Thus $(1+\omega_0) > 0$ represents a case of quintessence and $(1+\omega_0) < 0$ represents Phantom fluid dominated universe. Similar results are obtained by Hoyle and Narlikar [16-17] in C-field with negative kinetic energy for steady state theory of universe.
Now from equation (8) and (18) we get,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Omega_0}{2} H_0^2 \left[|\omega_0| + (1-|\omega_0|)\left(\frac{a_0}{a(t)}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}} \right]^{\frac{\alpha}{1+\alpha}} , \tag{24}$$

where $|\omega_0| = -\omega_0$, $H_0 = 100h\, km/s\, mpc$ present value of Hubble parameter and $\Omega_0 = \frac{\rho_0}{\rho_{cr,0}}$ with $\rho_{cr,0} = \frac{3H_0^2}{8\pi G}$.

Taking square root to both sides of equation (25) gives

$$\frac{\dot{a}}{a} = \sqrt{\frac{\Omega_0}{2}} H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}} \left[1 + \frac{(1-|\omega_0|)}{|\omega_0|}\left(\frac{a_0}{a(t)}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}} \right]^{\frac{\alpha}{2(1+\alpha)}} . \tag{25}$$

Expanding the R. H. S. of (25) and neglecting the higher powers of $\frac{(1-|\omega_0|)}{|\omega_0|}\left(\frac{a_0}{a(t)}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}}$ we get,

$$\frac{\dot{a}}{a} = \sqrt{\frac{\Omega_0}{2}} H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}} \left[1 + \frac{\alpha(1-|\omega_0|)}{2(1+\alpha)|\omega_0|}\left(\frac{a_0}{a(t)}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}} \right] . \tag{26}$$

Integrating (26) we get,

$$a(t) = \frac{a_0}{\left[2(1+\alpha)|\omega_0|\right]^{\frac{\alpha(1-2\xi_0)}{4(1+\alpha)}}} \left[\left(\alpha + 2(1+\alpha|\omega_0|\right) e^{8H_0|\omega_0|^{\frac{\alpha}{2(1+\alpha)}}\sqrt{\frac{\Omega_0}{2}}(t-t_0)} \right. \tag{27}$$

From equation (27), it is clear that as $t \to \infty$, $a(t) \to \infty$, therefore the present model is free from finite time future singularity.
In this case, the Hubble distance is given by,

$$H^{-1} = \frac{\sqrt{2}}{\sqrt{\Omega_0} H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}}} . \left[1 - \frac{\alpha(1-|\omega_0|)}{2(1+\alpha)|\omega_0|} . \left(\frac{a_0}{a(t)}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}} \right] . \tag{28}$$

Equation (28) shows the growth of Hubble distance H^{-1} with time such that $H^{-1} \to \frac{\sqrt{2}}{\sqrt{\Omega_0} H_0 |\omega_0|^{\frac{\alpha}{2(1+\alpha)}}} \neq 0$ as $t \to \infty$. Thus in present higher dimensional case, the galaxies will not disappear as $t \to \infty$, avoiding big rip singularity. Therefore, one can conclude that if Phantom fluid simultaneously behaves like generalized Chaplygin gas and fluid with $p = \omega\rho$ then the future accelerated expansion of the universe will free from catastrophic situation like big rip in higher dimensions.
Equation (18) can be written as

$$\rho = \rho_0 \left[|\omega_0| + (1-|\omega_0|)\left(\frac{a_0}{a(t)}\right)^{\frac{4(1+\alpha)}{\alpha(1-2\xi_0)}} \right]^{\frac{\alpha}{1+\alpha}} . \tag{29}$$

From equation (29), it is clear that as $t \to \infty$, $\rho \to \rho_0 |\omega_0|^{\frac{\alpha}{1+\alpha}} > \rho_0$.
Thus one can conclude that energy density increases with time, contrary to other phantom models having future singularity at $t = t_s$.

4. CONCLUSION

In higher dimensional FRW universe, we observed that when the cosmic dark energy behaves simultaneously like a fluid with equation of state $p = \omega\rho$; $\omega < -1$ as well as generalized Chaplygin gas with equation of state $p = -\frac{A}{\rho^{1/\alpha}}$, it is conclude that for $\dot{\phi}^2 > 0$, we get positive kinetic energy when $(1+\omega_0) > 0$ representing the case of quintessence and for $\dot{\phi}^2 < 0$, we get negative kinetic energy when $(1+\omega_0)<0$ representing phantom fluid dominated universe. The results are analogous to the results obtained by Hoyle and Narlikar in C-field with negative kinetic energy for steady state theory of universe. Further it is clear that as $t \to \infty$, $a(t) \to \infty$, therefore the present model is free from finite time future singularity. Also as $t \to \infty$, $H^{-1} \neq 0$ indicating that in present higher dimensional FRW universe, the galaxies will not disappear as $t \to \infty$, avoiding big-rip singularity. Therefore, one can conclude that when the cosmic dark energy behaves simultaneously like a fluid with equation of state $p = \omega\rho$; $\omega < -1$ as well as Chaplygin gas then big rip problem does not arise and the scale factor is found to be regular for all time. Also as $t \to \infty$, $\rho \to \rho_0 \left|\omega_0\right|^{\frac{\alpha}{1+\alpha}} > \rho_0$, concluding that energy density increases with time, contrary to other phantom models having future singularity at $t = t_s$. One should note that the results of Yadav [27] can be obtained from our results by putting appropriate values to the function concerned.

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